Linear Antenna Array synthesis with Decreasing Sidelobe and Narrow Beamwidth

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***Abstract*—Synthesizing arrays with low sidelobe and pencil beam radiation profile is under investigation for decades. A variety of array structures are available, but the simplest and useful structure is that of a linear array. Here, two basic symmetric Linear Antenna Array structures are assumed. The required array structure is assumed to provide low sidelobe and pencil beam profile. Departure from a uniformity in current and location profile has shown quiet appreciable improvement in the radiation pattern. The simulations are carried out using Differential Evolution Algorithm employing Best of Random mutation strategy (DEBoR).**

***Index Terms*—Non-uniform Current-Location profile, Low Sidelobe-Narrow Beam synthesis, Low Current Tapering, Differential Evolution Algorithm,Best of Random Mutation.**

I. Introduction

From past few decades since the concept of using arrays instead of a single element has been evolved, researchers have took the challenge to provide various array designs to provide a radiation characteristics according to the requirements [1-10]. Synthesizing an array depends on several matters, like requirements on the radiation pattern, directivity pattern etc. Radiation pattern depends on the number and the type of elements being used, the physical and electrical structure of the array etc. Numerous variations over the antenna structures as well as the type of elements are available, but for simplicity only one kind of elements are used in the whole array structure [11]. Aiming towards a radiation pattern with low sidelobe and narrow beam, this paper deals with linear array geometries with omnidirectional antenna elements without any inter-element phasing. For simplicity in formulation of pattern characteristics two symmetric structures of linear arrays are assumed, one without and the other with a centre element and the whole array is assumed to be broadside. The design goal in this paper is to find-out a probable profile of locating elements and the current distribution over the linear array aperture, that could provide low and decreasing sidelobes, narrow main beam, but with the least sacrifice in the directional characteristics. In search of a good radiation and directional characteristics, non-uniformity in both the current distribution and the inter-element distance profile is proposed. The establishment of the paper follows Design Equation in section II, A brief discussion on Differential Evolution Algorithm in section III, Discourse on results in section IV and ends up summing and concluding on the results in section V.

II. Design Equations

Figures 1 and 2 denote two Symmetric Linear Antenna Array structures placed along *z*-axis, without and with a centre element respectively. Symmetric linear broadside array structure without any centre element, have a generalized Array Factor as given by [11]

$$AF_w(I, x, \theta) = 2\sum_{n=1}^{N} I_n \cos(Kx_n \cos\theta) \tag{1}$$

The Array Factor corresponding to a symmetric structure as considered in the text with a centre element is given by,

$$AF_c(I, x, \theta) = I_0 + 2\sum_{n=1}^{N} I_n \cos(Kx_n \cos\theta) \tag{2}$$

Where, N is the number of elements on one side of the array axis,

I_n is the excitation amplitude of the n^{th} element from the centre, and $I = \{I_1, I_2, \cdots, I_n, \cdots, I_N\}$ is the current vector defining the current distribution over the aperture. Current vector is normalized to its maximum value. I_0 is the current amplitude for the centre element.

x_n is the distance of the n^{th} element from the centre, and $x = \{x_1, x_2, \cdots, x_n, \cdots, x_N\}$ is the location vector. Location vector is expressed in terms of the wavelength λ.

$K = \frac{2\pi}{\lambda}$ is the wave number, and θ is the azimuth angle.

DOI: 01.IJCOM.3.1.46

$L = 2x_N$ is the length of the array. The inter-element distance is assumed never to exceed the limit $(\lambda / 2, \lambda)$.

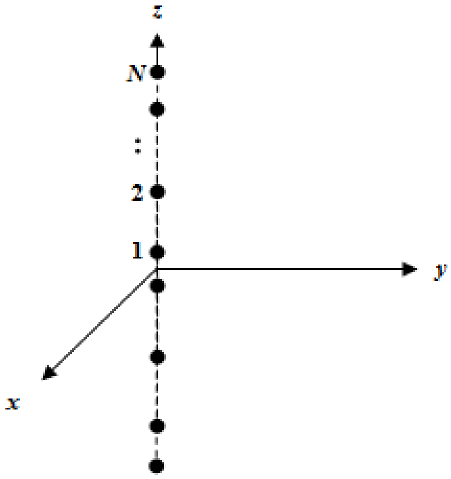

Figure 1. Schematic architecture of a symmetric Linear Array Antenna structure of *2N* elements placed along *z*-axis

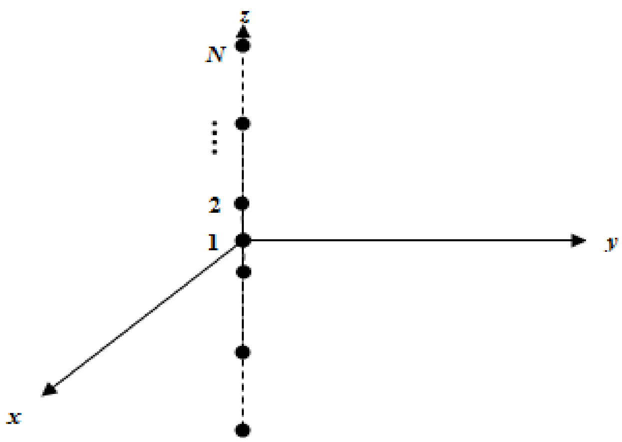

Figure 2. Schematic architecture of a symmetric Linear Array Antenna structure of *2N+1* elements placed along *z*-axis

The directivity of the non-uniform array is given by,

$$D_{NU} = \frac{\left(\sum_{k=1}^{N_{TOT}} \overline{I_k} \right)^2}{\sum_{l=1}^{N_{TOT}} \sum_{m=1}^{N_{TOT}} \overline{I_l}\,\overline{I_m} \frac{\sin\{K(z_l - z_m)\}}{K(z_l - z_m)}} \qquad (3)$$

Where, N_{TOT} is the total number of the element on the array aperture, $\overline{I_j}$ and z_j are the current amplitude and location of j^{th} element on the aperture looking the array from one end.

$N_{TOT} = 2N$, for a symmetric array without any centre element, and

$N_{TOT} = 2N + 1$, for a symmetric linear array with a centre element.

The Cost Function CF is designed carefully so as to shape the optimization as a minimization problem. It is given as

$$CF = \frac{SLL_U}{SLL_C} + \sum_k |AF(\theta_k)|^2 + TR + |BWFN_C - BWFN_U| \qquad (4)$$

Where,

SLL_C is the peak sidelobe level (SLL) of the design evolved with current generation in dB,

SLL_U is the SLL value in dB of the uniform array with the same length as found in the current iteration.

The term $\sum_k |AF(\theta_k)|^2$ is used to impose null [11] in every direction outside the main beam.

$BWFN_C$ denotes the first-null beamwidth (BWFN) of the radiation pattern with the parameters generated from current iteration,

$BWFN_U$ denotes the BWFN of the radiation pattern of the uniform array of same length as that of the non-uniform array generated in the current iteration.

III. The Differential Evolution Algorithm (DEA)

No Optimization technique has ever been superior on all problems. However, according to the survey by . Roscca, G. Oliveri and A. Massa [9], since Storn and Price had introduced a special type of Evolutionary Algorithm namely, Differential Evolution [6], a huge number of researches employing DEA have been carried out in the past decade, of which a significantly large portion were in the field of electromagnetics and antenna synthesis. In [8] a new improved variety of DEA is introduced, employing Best of Random selection strategy in mutation portion. This is called as Differential Evolution with Best of Random algorithm (DEBoR).

The main idea of DEA is to use vector differences in the creation of new probable solutions. DEA aims at evolving a population of S trial solutions to achieve a optimal solution in $K_{\max}$ iterations. Each of S individual is identified by one chromosome that codes a set of unknown descriptive of the problems solution. DEA proceeds in the following steps,

A. Initialization

S chromosomes each with N genes are generated. To generate a chromosome, z_n, $n = 1,2,\cdots N$ within its lower $z_n^{\min}$, $n = 1,2,\cdots N$ and upper $z_n^{\max}$, $n = 1,2,\cdots N$ bounds a uniform random string $r_{n,s}^{(k)} \in (0,1)$ is used in the following manner,

DOI: 01.IJCOM.3.1.46

$$z_{n,s} = z_{n,s}^{\min} + r_{n,s}\left(z_{n,s}^{\max} - z_{n,s}^{\min}\right)$$

Where r varies with $n, n = 1,2,\cdots N$, $s, s = 1,2,\cdots S$ and $k, k = 1,2,\cdots K_{\max}$. Cost Function $CF(z)$ is designed carefully for multi-objective optimization process. Maximum number of iterations $K_{\max}$, Scaling Factor F and Crossover probability CR is carefully initialized. F and CR are random variables in $0.4 \le F \le 1$ and $CR \in (0.3, 0.9]$.

B. Mutation

3 out of S chromosomes are randomly selected and recombined according to the Best of Random (BoR) strategy [8] to create a mutant vector v as below,

$$\underline{v}_s^{(k)} = \underline{z}_\alpha^{(k)} + F\sum_y \left(\underline{z}_{\beta_y}^{(k)} - \underline{z}_{\gamma_y}^{(k)}\right)$$

y is the number of the differential variations. α, β_y and γ_y are the indices of randomly chosen secondary parent $\underline{z}_\alpha^{(k)}$, and two randomly chosen donor vectors ($\underline{z}_{\beta_y}^{(k)}$ and $\underline{z}_{\gamma_y}^{(k)}$), of which,

$CF\left(\underline{z}_\alpha^{(k)}\right) \le \min\left\{CF\left(\underline{z}_{\beta_y}^{(k)}\right), CF\left(\underline{z}_{\gamma_y}^{(k)}\right)\right\}$ z_s^k is called the primary parent.

C. Crossover

Binary Crossover is adopted for generating new offspring solution vector $u_{n,s}^{(k)}$ to compete with the corresponding primary parent z_s^k. The process is done as

$$u_{n,s}^{(k)} = \begin{cases} v_{n,s}^{(k)} & if\ r1_{n,s}^{(k)} < CR \\ z_{n,s}^{(k)} & otherwise \end{cases},$$

Where, $r1_{n,s}^{(k)}$ is a string of uniformly distributed random numbers over the range $(0,1)$. is kept under the same bounds as specified for .

D. Selection

Based on the fitness is operated on and in order to keep better solutions for the next iteration and discard the relatively poor one.

$$\underline{z}_s^{k+1} = \begin{cases} u_s^{(k)} & if\ CF\left(\underline{u}_s^{(k)}\right) < CF\left(\underline{z}_s^{(k)}\right) \\ z_x^{(k)} & if\ CF\left(\underline{z}_s^{(k)}\right) \le CF\left(\underline{u}_s^{(k)}\right) \end{cases}$$

The steps namely Mutation, Crossover and Selection are continued until the maximum iteration is reached.

IV. Results and Discussions

This segment establishes the modelled results for different broadside symmetric linear antenna array designs optimized by DEBoR profiency. Eight groups of linear array structures are studied. Sets for harmonious structures without core elements include 14, 18, 22 and 26 elements, while for centred elemented symmetric structures are taken as 15, 19, 23 and 27. It is found that the best conclusions are received for an starting population of 120 chromosomes. Utmost number of generations is limited to 400. Scaling Factor F for mutation is selected as 0.49. Binary Crossover is chosen with Crossover Probability CR as 0.35.

A. Numerical Data and Resultant Patterns

Parameters related to the physical, electrical and directional properties are Tabulated in Table I, II and III. Inter-element distance for a uniform array and Location of elements for a non-uniform array is considered as physical parameter, while excitation amplitude of each element pair is considered as the electrical parameter. Current amplitude for the uniform array is considered to be 1. Radiaton parameters are considered as the SLL and BWFN. Directivity is commemorated for the non-uniform array as radiation parameter. Table I records the SLL and BWFN and Peak Directivity of the array sets. The initial inter-element spacing is considered as $\frac{\lambda}{2}$. For symmetric linear array sets without and with centre element Table II and III record the location of element pairs and the respective current amplitudes and compares the SLL and BWFN with that of the representing consistent array with same length. Peak Directivity as proposed by the optimal non-uniform set is marked in each of the Tables .

Figures 3 and 4 equates the radiation patterns of non-uniform array groups to their representing equally long uniform array sets for 26 and 27 elements respectively. Radiation patterns are diagrammed in linear dB scale in θ-plane, since linear arrays as regarded in these papers execute omni-directional properties in ϕ-plane.

Table I. SLL, BWFN, and Directivity for Uniform Linear Array Sets with inter-element spacing as $\lambda/2$

Set No.	Number of Elements	SLL (dB)	BWFN (°)	Directivity
Ia	14	-13.30	16.43	14
Ib	15	-13.32	15.33	15
IIa	18	-13.37	12.76	18
IIb	19	-13.38	12.09	19
IIIa	22	-13.40	10.43	22
IIIb	23	-13.40	9.98	23
IVa	26	-13.42	8.82	26
IVb	27	-13.42	8.50	27

DOI: 01.IJCOM.3.1.46

TABLE I. LOCATION OF ELEMENTS, CURRENT APLITUDES, SLL, BWFN, AND DIRECTIVITY FOR NON-UNIFORM SYMMETRIC LINEAR ARRAY SETS WITHOUT CENTRE ELEMENT

Set No.	Location of Elements	Current Amplitudes $I_1, I_2 .. I_N$	SLL (dB)		BWFN (°)		D_{BNU}
			Opt.	Unif.	Opt.	Unif.	
Ia	±0.26 ±0.82 ±1.36 ±2.05 ±2.87 ±3.72 ±4.46	0.90 0.78 0.96 0.97 1.00 0.91 0.75	-18.27	-13.30	13. 50	11.94	14.49
IIa	±0.28 ±0.82 ±1.42 ±2.04 ±2.69 ±3.47 ±4.32 ±5.16 ±5.90	0.91 0.88 0.86 1.00 0.95 0.85 0.84 0.74 0.69	-20.63	-13.37	11.01	9.18	18.63
IIIa	±0.24 ±0.74 ±1.24 ±1.75 ±2.25 ±2.81 ±3.36 ±4.04 ±4.86 ±5.70 ±6.44	0.78 0.95 1.00 0.89 0.83 0.95 0.92 0.93 0.81 0.76 0.73	-21.13	-13.40	10.43	8.50	22.82
IVa	±0.28 ±0.81 ±1.37 ±1.92 ±2.49 ±3.08 ±3.66 ±4.33 ±4.98 ±5.66 ±6.43 ±7.25 ±7.99	0.97 1.00 0.91 0.90 0.90 0.85 0.93 0.90 0.78 0.70 0.73 0.75 0.68	-22.02	-13.42	8.49	6.90	26.97

TABLE I. LOCATION OF ELEMENTS, CURRENT APLITUDES, SLL, BWFN, AND DIRECTIVITY FOR NON-UNIFORM SYMMETRIC LINEAR ARRAY SETS WITH A CENTRE ELEMENT

Set No.	Location of Elements	Current Amplitudes $I_0, I_1, I_2 .. I_N$	SLL (dB)		BWFN (°)		D_{BNU}
			Opt.	Unif.	Opt.	Unif.	
Ib	0 ±0.50 ±1.01 ±1.53 ±2.17 ±2.94 ±3.77 ±4.49	1.00 0.95 0.88 0.96 0.91 0.88 0.80 0.67	-19.35	-13.32	15.17	11.94	15.13
IIb	0 ±0.57 ±1.08 ±1.63 ±2.25 ±2.85 ±3.54 ±4.37 ±5.18 ±5.93	1.00 0.92 0.89 0.88 0.84 0.88 0.88 0.82 0.78 0.68	-20.35	-13.38	11.09	9.16	19.91
IIIb	0 ±0.50 ±1.01 ±1.51 ±2.02 ±2.64 ±3.21 ±3.92 ±4.70 ±5.55 ±6.41 ±7.11	1.00 0.84 0.88 0.74 0.92 0.93 0.85 0.91 0.89 0.77 0.75 0.67	-21.56	-13.40	9.53	7.71	23.69
IVb	0 ±0.53 ±1.07 ±1.59 ±2.15 ±2.80 ±3.56 ±4.27 ±4.86 ±5.55 ±6.34 ±7.18 ±8.06 ±8.78	1.00 0.87 0.80 0.78 0.90 0.99 0.90 0.94 0.77 0.79 0.87 0.74 0.74 0.68	-20.23	-13.42	7.46	6.29	27.07

Figures 5 and 6 portray that an adept selection of position of elements and current distribution over the array aperture can remarkably mend the resultant radiation pattern, even if compared to equally long uniform array. The results are tabulated in Tables I, II and III.

From the Tables I and II and Figure 3 it is clearly viewed that, for a 26 element linear array, radiation pattern of both the uniform arrays have -13.42 dB whereas, the non-uniform array inhibits the sidelobe to -22.02 dB. The Directivity of the non-uniform array is 26.97 and that of the initial array is 26. The BWFN of the non-uniform array is 8.49°, and those for initial and equally long uniform arrays are 8.82° and 6.90°.

Figure 4 and the Tables I and III show that for 27 element array the SLL value of the non-uniform array is reduced to -20.23 dB against -13.42 dB of the uniform arrays. Directivity of the non-uniform array is 27.07 while that for the initial array was 27. BWFN of the optimized set is 7.46°, while those for the initial and equally long uniform arrays are 8.50° and 6.29° respectively.

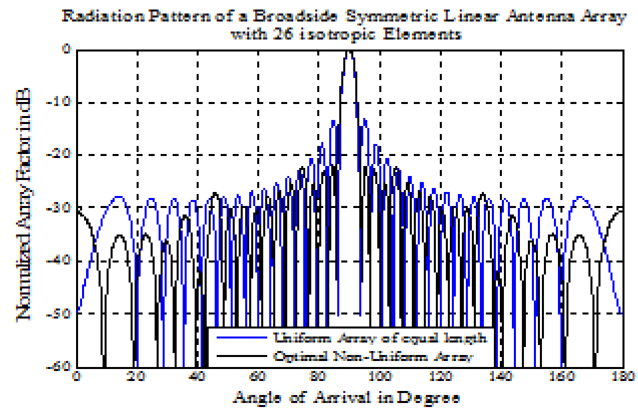

Figure 3. Θ-Plane radiation Patterns of 26 element broadside symmetric linear antenna arrays

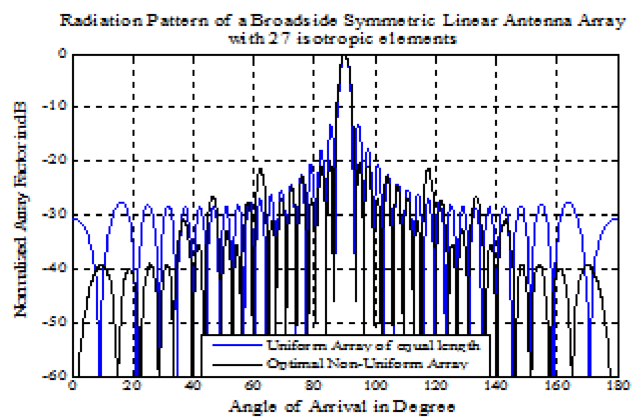

Figure 4. Θ-Plane radiation Patterns of 27 element broadside symmetric linear antenna arrays

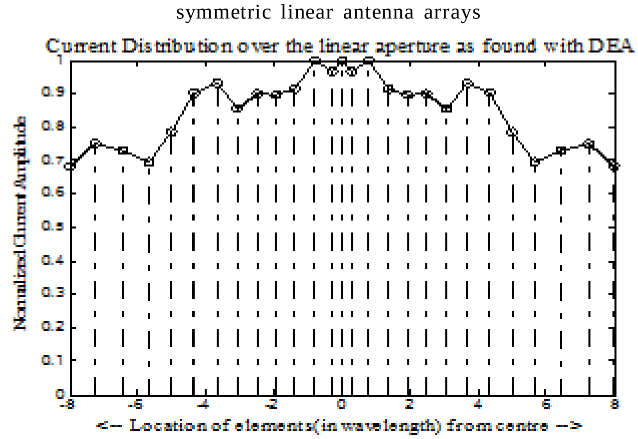

Figure 5. Optimal Current distribution over the array aperture of 26 element broadside linear antenna array

DOI: 01.IJCOM.3.1.46

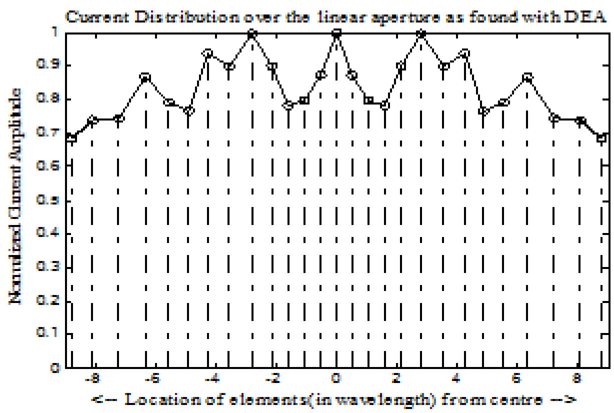

Figure 6. Optimal Current distribution over the array aperture of 27 element broadside linear antenna array

B. Convergence Profile of DEBoR

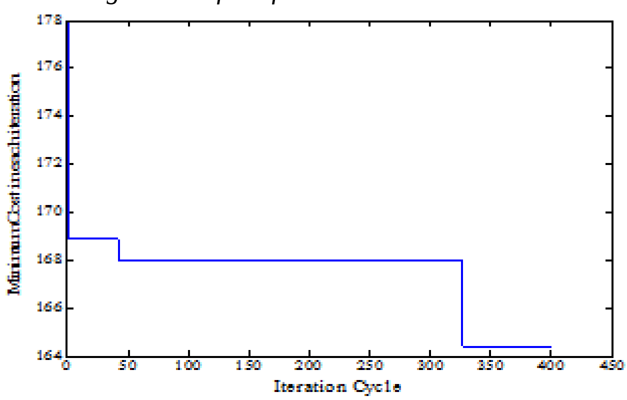

Figure 7. Convergence Profile of Differential Evolution Algorithm for optimizing the 26 element broadside symmetric linear array set

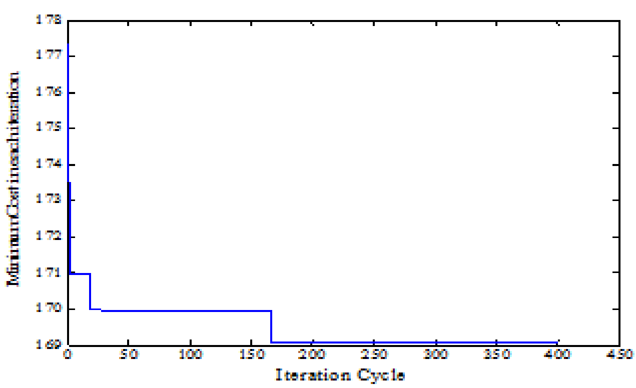

Figure 8.Convergence Profile of Differential Evolution Algorithm for optimizing the 27 element broadside symmetric linear array set

Figures 7 and 8 registers the convergence profiles of Differential Evolution Algorithm as the respective optimization processes for 26 and 27 element arrays proceeded. Minimum value of the Cost Function CF in each iteration is recorded for the respective optimization process. These curve display that the best up to the current iteration in each process is saved and thus results are prohibited from deterioration. The optimization processes for 26 and 27 elemental arrays are found converging after 337 and 167 iterations. The programming has been written in MATLAB language using MATLAB 7.5 on core (TM) 2 duo processor, 1.83 GHz with 2 GB RAM.

Conclusions

The modelled resultants have demostrated that considerable meliorations can be detected with the current-location non-uniformity. Less dwindling of currents have rendered overall sidelobes in reducing manner and the main-beamwidth in between the initial and equally long uniform arrays. Thus, it defeats the problems due to unceasing sidelobes in a large and the peak directivity is restrained to be leastwise to that of the initial array. Hence, Differential Evolution with Best of Random mutation scheme has executed successfully to work on the linear array synthesis problem.

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DOI: 01.IJCOM.3.1.46